MATH 2650 Optimal Control Theory(3 credit hours)

Elmira College

SPRING 2025

Required Text:
D. Liberzon. (2012) ***Calculus of Variations and Optimal Control Theory: A Concise Introduction***, Princeton Univ.
Rawlings, J. B., Mayne, D. Q., & Diehl, M. M. (2020). ***Model Predictive Control: Theory, Computation, and Design (2nd ed.)***. Nob Hill Publishing.

Pre-requisites: MATH 2520 Principles of Linear Algebra and Differential Equations

Course Description

Optimal Control Theory is a branch of applied mathematics that deals with finding control strategies to optimize the performance of dynamic systems. This course is delves into fundamental principles, the Euler-Lagrange equation, variational problems subject to constraints, and provides illustrations of control systems. Students will develop a strong theoretical foundation and gain hands-on experience through computational exercises and projects.

Course Objectives and Goals

- Understand the mathematical formulation and solution methods for optimal control problems.
- Master key concepts such as the calculus of variations, Pontryagin's Maximum Principle, dynamic programming, and linear quadratic regulators.
- Apply optimal control techniques to solve real-world problems involving dynamic systems.
- Explore advanced topics like stochastic control, model predictive control, and reinforcement learning.
- Develop proficiency in using computational tools for optimal control analysis and design.

Evaluation of Performance

Your grade will be based upon your performance on exams, assignments, and participation.

2 Assignments	10%
4 Quizzes	30%
Midterm Exam	15%
Final Project	25%
Final Exam	20%
Total	100%

Grades will be assigned as follows:

A	93% and above	B-	80 - 82%	D+	67 - 69%
A-	90 - 92%	C+	77 - 79%	D	63 - 66%
B+	87 - 89%	C	73 - 76%	D-	60 - 62%
B	83 - 86%	C-	70 - 72%	F	59% or below

Withdrawal Policy: Please see Elmira College Bulletin for information on this policy.

Academic Honesty: Please read the section on Academic Honesty in the **Code of Conduct**. Briefly, academic dishonesty includes: cheating, fabrication, facilitating academic dishonesty, and plagiarism. Ask if you have any questions on whether something constitutes as academic dishonesty. All work must be original and new. Past assignments from current or other courses will not be accepted. Academic dishonesty will not be tolerated. It will result in zero on the assignment, and a report will be filed with the school. Continued practice will result in failure of the class. Institutional penalties may also apply with repeated acts of academic honesty.

Student Responsibility:

- It is your responsibility to keep track of assignments and due dates.
- You should ask questions concerning assignments and lectures, if you need any clarifications.
- If you are struggling in class, have concerns, and/or unsure about expectations, please stop by during office hours or make an appointment for another time.

Tentative Schedule of Topics

Topic	Modules	Tasks & Evaluations
Introduction to Optimal Control ● Overview of optimal control problems. ● Path optimization vs. point optimization. ● Basic concepts and terminology.	Module 1	
Static Optimization ● Unconstrained optimization problems, constrained optimization problems.	Module 2	
Optimal Control of Discrete-Time Systems ● Formulation, discrete-time linear quadratic regulator, digital control of continuous-time systems, steady-state control and suboptimal feedback.	Module 3	Assignment 1
Calculus of Variations ● Euler-Lagrange equation and its applications. ● Hamiltonian formalism and Legendre transformation. ● Constraints, weak and strong extrema, and conjugate points	Module 4	
From Calculus of Variations to Optimal Control ● Control system, cost functional, target set.	Module 5	Quiz 1
Pontryagin's Maximum Principle	Module 6	

• Statement and proof of the maximum principle. • Bang-bang and singular controls. • Relation to Lie brackets and geometric control		
Dynamic Programming and HJB Equation • Dynamic programming and Bellman's principle of optimality. • Hamilton-Jacobi-Bellman (HJB) equation and viscosity solutions	Module 7	Quiz 2
Linear Quadratic Regulator (LQR) • Problem Formulation: Linear systems with quadratic cost functions. • Riccati Equation: Derivation and solution methods. • Feedback Control: Optimal state feedback laws.	Module 8	Midterm Exam
Linear Quadratic Gaussian (LQG) Control • Stochastic Systems: Optimal control under uncertainty. • Kalman Filter: State estimation for linear systems. • Separation Principle: Combining LQR and Kalman filter.	Module 9	
Linear Quadratic Gaussian (LQG) Control • Stochastic Systems: Optimal control under uncertainty. • Kalman Filter: State estimation for linear systems. • Separation Principle: Combining LQR and Kalman filter.	Module 10	Assignment 2
Model Predictive Control (MPC) • Receding Horizon Control: Optimization over a finite horizon. • Constrained Optimization: Handling state and control constraints. • Computational Methods: Quadratic programming and real-time implementation.	Module 11	Quiz 3
Stochastic Optimal Control • Stochastic Differential Equations: Modeling uncertainty. • Expected Cost Optimization: Minimizing the expected value of the cost function. • Stochastic HJB Equation: Extensions of the HJB equation.	Module 12	
Optimal Control for Nonlinear Systems • Nonlinear Dynamics: Challenges and techniques for nonlinear systems. • Feedback Linearization: Transforming nonlinear systems into linear ones.	Module 13	Quiz 4

• Lie Algebra and Geometric Control: Advanced tools for nonlinear control.		
Sliding Mode Control • Sliding Surfaces: Designing sliding surfaces for robust control. • Sliding Mode Dynamics: Properties and stability analysis.	Module 14	
Optimal Stopping Problems • Bruss's odds algorithm • Optimal stopping over the infinite horizon	Module 15	Final Project
Advanced Applications and Case Studies • Case Studies: Real-world applications in aerospace, robotics, and economics. • Multi-Agent Systems: Optimal control of distributed systems. • Hybrid Systems: Optimal control of systems with both continuous and discrete dynamics. • Future Directions: Emerging trends and open problems in optimal control.	Module 16	Final Exam